The ‘Giant Steps’ Progression and Cycle Diagrams

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One of the challenges faced by any intermediate Jazz student is to master the *Coltrane changes* in every key. The term *Coltrane changes* refers to the harmonic progression associated with the tune ‘Giant Steps’ by John Coltrane. Besides using this progression as the basis of ‘Giant Steps’, Coltrane also applied these changes as a *substitute* pattern over the chord changes of a number of standard tunes to which he composed new melodies, such as Countdown (based on Tune Up), Sattelite (based on How High The Moon), 26-2 (based on Confirmation). In some cases (Body and Soul, But Not For Me) Coltrane applied the same changes as substitute patterns for turnarounds using slight modifications of the original melodies. Clearly, Coltrane viewed this chord progression as a formula that can be applied in many contexts and in many keys.

Most approaches to learning the Coltrane harmonic progressions are based on memorization, and when faced with memorizing a long chord sequence in 12 keys – most people find the task daunting. However, you will see in this article that the ‘Giant Steps’ harmonic progression is a *musical cycle*, and like all musical cycles is based on easy to memorize formulas.

I started to investigate musical cycles systematically following a master class by the great alto saxophonist *Gary Bartz* at Michiko Rehearsal Studios in NYC, where he made the statement that all the secrets of Jazz harmonic progressions and substitutions can be deduced from studying cycles. I was fascinated by this idea that everything from II-V-I patterns to Tritone substitutions to *Coltrane changes* can all be deduced from a simple set of rules, and so I started down the path of understanding the principles behind *cycles*.

Basic Cycle Math

A *musical cycle* is an ordered set of notes obtained by successively applying the same interval. In other words: take any interval from a starting note, find the next note, take the same interval from that note, and so on. When do you stop? When you reach the same note you started out with. Hence the name *cycle*.

Lets look at some simple math relating to cycles. First of all, there are 12 semi-tones (notes) in an octave. On the other hand, interval numbers are based on the location of the note in the 7-note major scale. We think of an 8^{th} interval as an octave, and we use *major* and *minor* (or *augmented* and *diminished*) to modify the intervals. Thus, a half step is called a minor 2^{nd} (m2), and a whole step is called a major 2^{nd} (M2).

Now, you probably already know the cycle of fifths, so you know that you can read that cycle as 5^{th}s in one direction and as 4^{th}s in the other direction. What’s the general principle here? Any cycle will have the property that if you read it one-way or the other – the intervals will add up to 9 (e.g. for the cycle of fifths: 4+5 = 9). There are 12 semi-tones, and a major scale octave, why do the intervals then add up to 9 instead of 7 (7 being the number of intervals between 8 scale notes)? Before you read on, you might want to stop for a moment and try to figure this out yourself.

The answer is that we count intervals in ordinal numbers starting from 1 (unison). If we were counting the spaces between the notes, we would count a unison as 0 rather than 1, and a 2nd as 1 – but since we count in this way in both directions – we add one from both directions and end up with 9 instead of 7. Following is one octave with the scale intervals written above (counting up) and below (counting down) and you can see that the upper number and the lower always add up to 9.

1 2 3 4 5 6 7 8

C D E F G A B C

8 7 6 5 4 3 2 1

If you expand this to include all 12 notes, you will notice that the numbers still add up to 9, and that one is always M (major) and the other is m (minor), or both are the same (the tritone) or both are perfect (4th and 5th unison and octave).

1	m2	M2	m3	M3	4	TT	5	m6	M6	m7	M7	8	(up)
C	C#	D	Eb	E	F	Gb	G	Ab	A	Bb	B	C	
8	M7	m7	M6	m6	5	TT	4	M3	m3	M2	m2	1	(down)

Now that we've got that mystery solved, lets get back to cycles. In order to systematically cover all possible cycles, lets walk through all of the interval combinations above and below each note and see what cycles each one yields. So, here's the second puzzle for you to think about before reading on: How many possible musical cycles are there?

The answer is that there are only 7 musical cycles. If you look at the sequence above again, you will notice that we only have to walk half way left-to-right up to the tritone, because from that point onwards the interval pairs simply get repeated (inverted) and since a cycle goes in both directions – we will have already included them. So, there are only 7 possible cycles in all of music. That's pretty amazing if you think about it.

The next interesting point to think about is how many notes are there in each cycle before we get back to the same note? Well, that varies from cycle to cycle. The maximum is, obviously 12, and the minimum is clearly 1. What do you do when there are less notes in the cycle than 12? You have multiple instances of the same cycle.

Enumerating the cycles

Lets enumerate all 7 possible cycles and see what we can learn from them. For simplicity, I will start with the 2 cycles that encompass all 12 notes: the m2/M7 cycle (known as the *chromatic cycle*) and the 4/5 cycle (known as the *cycle of fifths*).

The first cycle is the chromatic cycle. Walking clockwise, you get m2 intervals, and counter clockwise you get M7 intervals. I am only showing flats since those are more common as root movements in Jazz – but you should also think of the equivalent sharp notes. Hopefully, you already know this cycle by heart, so there is nothing here to learn.

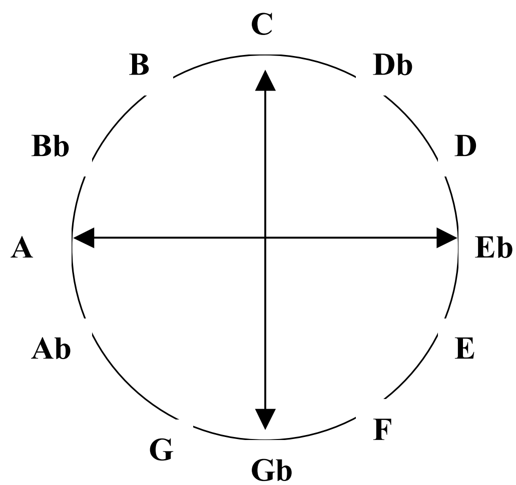

Figure 1: Chromatic Cycle (m2/M7)

The diagram below shows the cycle of fifths. In this cycle, if you walk clockwise, you will be moving perfect 4th each step, and if you walk counter-clockwise, you will be moving a 5th each step.

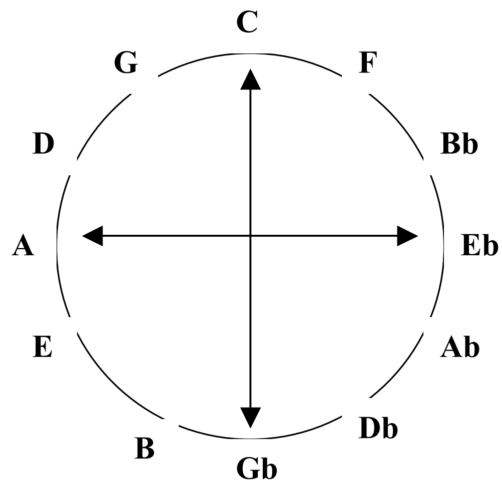

Figure 2: Cycle of Fifths (P4/P5)

It's impossible to overstate the importance of the cycle of fifths in Jazz. In fact, if you start from any note and walk clockwise 2 notes, you are basically covering a II-V-I sequence. If you walk 4 notes – you are covering a III-VI-II-V chord sequence. If you walk clockwise 6 notes, you get a sequence that is typically harmonized as bVm7b5 – VII7 and then proceeds down to III-VI-II-V. You can use this cycle as the basis for many reharmonizations by picking a target chord and "back-cycling" to it from any point that is up to a tritone away (diagonal) on the cycle of fifths.

Did you notice some amazing symmetries between the chromatic cycle and the cycle of fifths? The first thing to notice about both of these cycles is that the *tritone* (Gb or F#) is in the middle of both cycles at the 6 o'clock position. The tritone divides the octave in half. And you can get to it from the root by taking 6 succesive minor 2nds, 6 successive Major 7ths, 6 successive perfect 4ths, or 6 successive perfect 5ths.

In fact, if you trace the diagnoal between any two notes that are across from each other – they form a tritone interval. This is true of *both cycles*! No wonder people called the tritone the *devil's interval*!

As if that's not surprising enough, did you notice that in both cycles, C, Eb, Gb and A are in the same exact spot at the 12, 3, 6 and 9 o'clock positions? That works out because going up three 4ths is a minor 10th which is equivalent to a minor 3rd.

You can take the symmetry even further: take every other note in the cycle of 5ths and flip it with its tritone (which is diagonally across) – you get the chromatic cycle, and vice versa. But then, you already knew that, didn't you? It's the tritone substitution! If you take a II-V-I and flip the V with its tritone you get II-bII7-I which is a chromatic progression. Similarly, III-VI-II-V becomes III-bIII7-II-bII. So, the chromatic cycle and the cycle of fifths are related through the tritone substitution.

Five more cycles to go. Lets get the trivial one out of the way: the unison/octave cycle. This is really a degenrated cycle of one. Each note is its own unison and its own octave, so we end up with 12 possible cycles of 1 note each. Pretty useless. So, lets ignore it.

The next one we will look at is the tritone cycle. In this cycle, the intervals going both ways are the same, so this results in only two notes per cycle. No point in depicting that as a circle, so lets depict it as a line connecting the two notes. We can see that there are 6 instances of this cycle needed to cover all 12 notes:

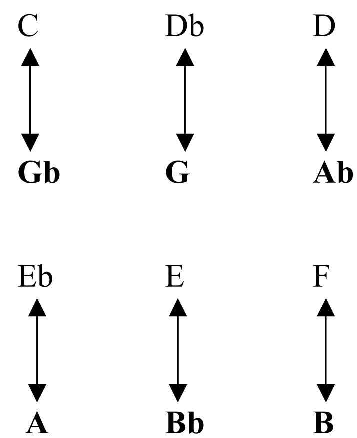

Figure 3: Cycle of Tritones (6 instances of b5 pairs)

So, there are only 6 different tritone pairs to memorize. The tritone interval is very important in Jazz as it represents one of the most common substitutions. Typically, this is notated as substituting a V7 chord with a bII7 – however, this way of thinking about it would lead you to believe that you need to memorize 12 such pairs, when in fact there are only 6 pairs when you think of them as cycles.

The fifth cycle to consider is the M2/m7 cycle (also known as the *whole tone cycle*). There are only two whole tone cycles each containing 6 notes, so we can arrange them as two *hexagons*. Notice that the tritone appears in this cycle as well: each two notes across from each other are a tritone apart (hence the name: tritone = 3 whole tones). We already saw that this was also true of the chromatic cycle and the cycle of fifths.

Another important thing about this cycle is that if you walk it counter-clockwise the intervals are minor 7ths. Also, think about how you could super impose the two whole tone cycles to get the chromatic cycle.

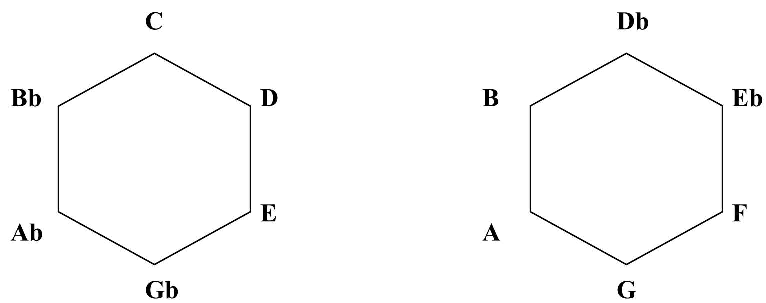

Figure 4: The Whole-Tone Cycles (2 instances of M2/m7)

We have two more cycles left to cover: m3/M6 and M3/m6: these correspond to the diminshed and augmented 3rd intervals.

The Diminished Cycle consists of m3 intervals going clockwise, and M6 intervals going counter clockwise. There are 3 instances of the cycle, each representing a Dim7 chord. Did you ever wonder why you can move a Dim7 chord up a minor 3rd and still get the same chord? The answer is that you are "stuck" in one of these 3 cycles and anywhere you start and walk clockwise you will be stacking up three minor 3rds in a row to create a Dim7 inversion. Since there are 4 notes per cycle, we can visualize it as a square. Guess what? The diagonals are tritones again (2 minor 3rds = tritone).

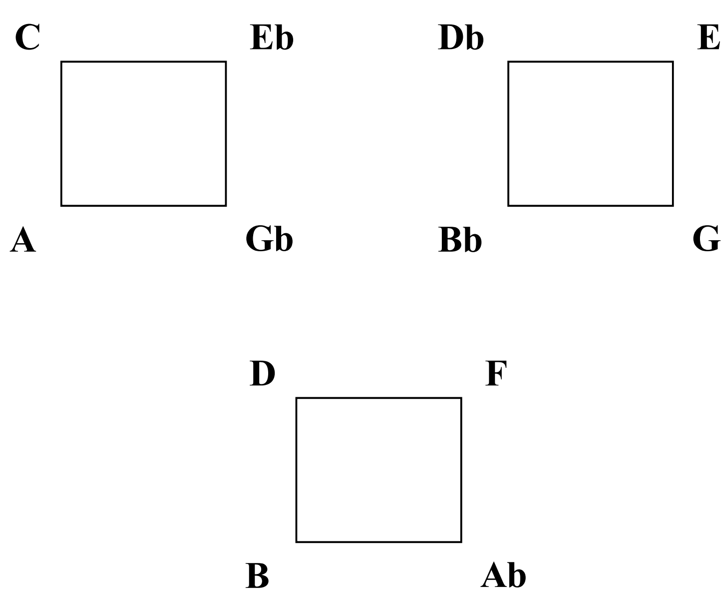

Figure 5: The Diminished Cycle (3 instances of m3/M6)

The *Diminished Cycle* was also used by Coltrane and others as a formula for root movements. The first three chords of ‘Like Sonny’ progress according to this cycle: Dm7-Fm7-Abm7. It was also used as a turnaround in Joe Henderson’s blues ‘Isotope’ using an ascending formula in C: A7-Gb7-Eb7-C7. Combinations of the diminished cycle, the cycle of fifths and the chromatic cycle appear in many turnarounds and reharmonizations. For example, the turnaround for “Lady Bird” is: CM7-EbM7-AbM7-DbM7(-CM7), which combines movements from all three cycles.

The ‘Giant Steps’ Cycle

Finally, we arrive at the last and most intriguing cycle: The *Augmented Cycle* based on the M3/m6 formula, which gets us back to ‘Giant Steps’. The major 3rd interval creates four possible cycles of 3 notes each. Moving clockwise by major 3rds and counter clockwise by minor 6ths. Again, since we are discussing cycles, I am showing enharmonic flats, even when the correct notation for many of the M3 intervals would be sharps.

You can view each of these cycles as an Augmented chord, which once again sheds some light on the fact that there are only 4 different augmented chords (meaning that you can move any augmented chord up or down a major 3rd). Also, you can see that a minor 6th is equal to two major 3rds (since you can get to any note by either taking 2 clockwise steps of M3 or 1 counter clockwise step of m6). Another thing to notice is that in a triangle there are no diagonals, and indeed this is the only cycle in which the tritone does not appear.

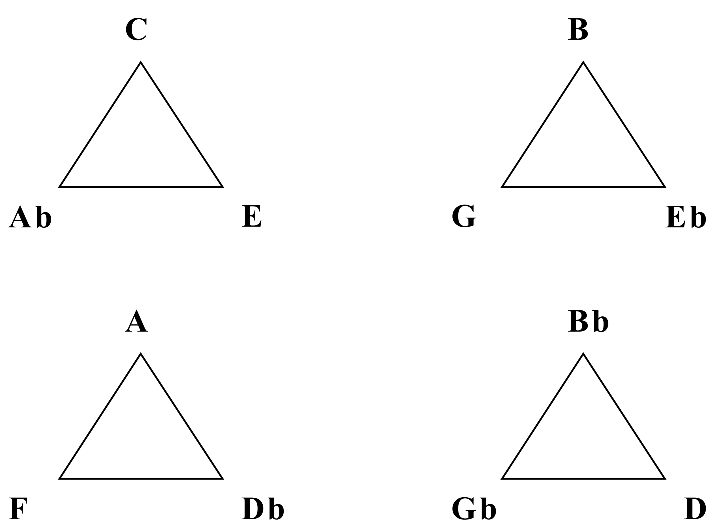

Figure 6: The Augmented Cycle (4 instances of M3/m 6)

Coltrane used these 4 cycles as the basis for the root motion of ‘Giant Steps’ – which brings us back to why we started looking at these cycles in the first place. In fact, the top right triangle is the entire formula of the tonal centers of ‘Giant Steps’ in the original key.

We can expand this slightly by merging in one piece of extra information from the cycle of the fifths: precede each root by its V7. This leads to two possibilities of traversing the triangle: walk to the left or walk to the right.

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Right:  B - Bb7 | Eb - D7 | G  - F#7 | B ||
Left :  B - D7  | G - Bb7 | Eb - F#7 | B ||  (Giant Steps)
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My guess is that Coltrane chose the 2nd option because adding the V7 before the roots in the first instance created a ½ step downward bass motion, which makes the progression sound more predictable, since root motion from I to VII7 is a common vehicle for going to the diatonic III and this progression would then sound like a standard major/minor deceptive cadence (going to IIImaj7 instead of IIIm7).

How does all of this help you master the 'Giant Steps' progression in all keys? The answer should be obvious by now. All you need to remember are the 4 triangles of augmented root motion and precede each one by its V7 chord.

What about 'Countdown'? Well, it turns out that it's only a slight variation on the same idea. Instead of the I chord in the first bar – just use the IIm7. The rest stays exactly the same. Lets take it from the bottom right triangle and make the tonal center start with D.

The original 'Tune Up' progression is simply II-V-I in D:

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Em7  | A7 | Dmaj7 | Dmaj7 ||
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Lets look at Giant Steps and Countdown variations in D:

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GS :    D - F7  | Bb - Db7 | Gb - A7 | D ||
CD :  Em7 - F7  | Bb - Db7 | Gb - A7 | D ||
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As you can see the only difference is that we started with the IIm7 chord in D rather than the I. The progression then repeats in C and Bb tonal centers. Pretty simple, huh? You don't really have to memorize a complicated progression in 12 keys – you just have to memorize the 4 triangles, which you already know, since they are just the 4 distinct augmented triads.

It is interesting to note that the augmented cycle was used by Rogers & Hart in the bridge of 'Have You Met Miss Jones' – but it wasn't until Coltrane that people started thinking of it as a reusable chord progression and a reharmonization device. Since we have now exhausted the set of possible pure cycles to use in progressions, any new chord progressions invented in the future will have to mix intervals from multiple cycles.

Hybrid Cycles and Scale Diagrams

What would happen if we expand the definition of a cycle to allow for more than one type of interval? At this stage, we are venturing into the realm of *scales* more than *cycles*.

However, it turns out that organizing the notes of a scale in a *cycle diagram*, as we did for the 7 “pure’ cycles, can be very useful as a learning tool because it helps you spot some relationships that you might otherwise miss. I also believe that this helps you visualize the scale similar to the way piano players do – as a continuum of consonant notes across multiple octaves.

Lets start with the diminished scale (the whole-half step scale). That is an 8-note scale, which can be easily constructed from the dimished cycle by adding a whole step after each of the original notes.

As you look at these 3 scales in cycle format you can see some interesting things that are harder to visualize on an instrument. Since there are 3x8=24 notes in these 3 scales – each note appears twice. This corresponds to the half-whole and whole-half step scales. For example, if you start from C on the top-left cycle – you get the whole-half scale, and if you start from C on the top-right cycle you get the half-whole. In fact, this means that you really only have to learn 3 versions of this scale. The second thing to notice is that our good friend the tritone is here as well: any two notes diagonal from each other are a tritone apart.

Another interesting thing to notice is that if you draw a triangle as I show on the bottom scale – you get a major triad. In fact, if you spin that triangle around – you will keep getting major triads.

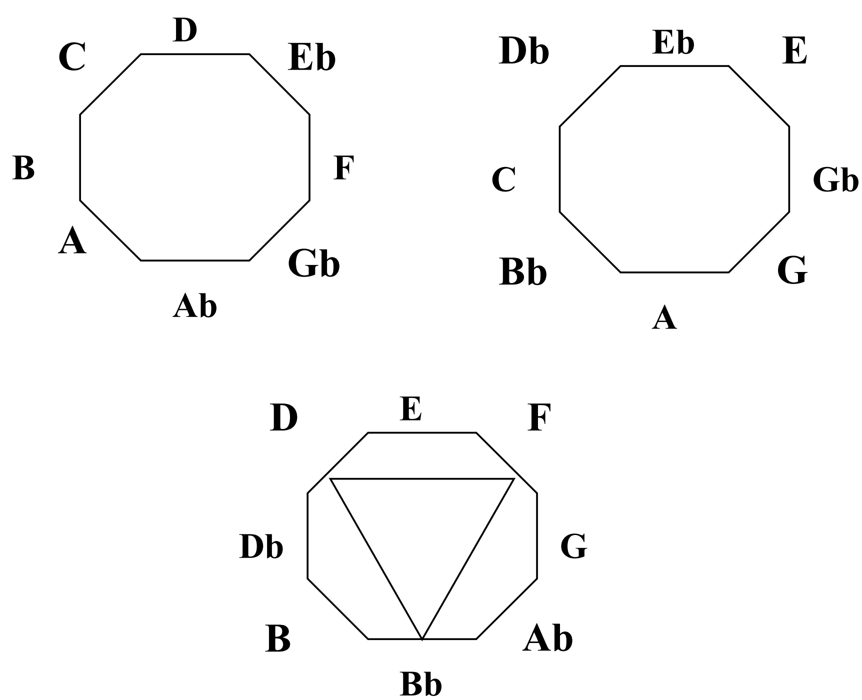

Figure 7: The 3 Diminished Scales/Cycles

Finding triads and other hidden structures within these scales/cycles is what makes this way of visualizing them useful. You can sit with your instrument in front of these diagrams and work out triads, four-note chords, voicings and lines.

Using Cycle Diagrams to master Scales

Based on this idea, I find that it is very useful to create *cycle diagrams* even for the basic scales and use them as a visualization aid while you are learning voicings and diatonic exercises. When used in this way, you would create a separate diagram for each scale in each key based on the degrees of the scale. For 7-note scales we use a *heptagon*:

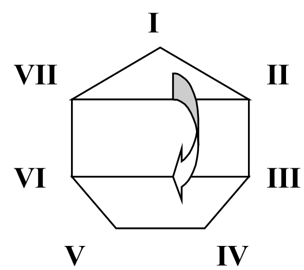

Figure 8: Cycle Diagram Formula for 7-note Scales

To make this more concrete, lets look at two sample diagrams for major and melodic minor scales in the key of C:

Figure 9: C Major and Melodic Minor Cycle Diagrams

You can use this cycle diagram in many different ways. For example, you can use it to find all triads, 7th chords and tensions by simply starting from the scale degree of interest and then move clockwise skipping every other note. You can work on all the modes of each scale from this diagram as well. For example, if you start walking the C melodic minor diagram clockwise from F you will get the Lydian dominant scale.

You can also use this diagram as a reference to work on moving voicings up and down diatonically – a technique used frequently in modern jazz comping. Take a 3-note voicing such as C-D-G and move each note clockwise to get to the next voicing.

Another idea is to use this diagram as a guide for working on melodic sequences. Start simple with 3rds by skipping every other note and work through the various intervals by counting up from each start note. Next, you can pick a scale pattern such as: 1235 (C-D-E-G) and move it along the diagram. You can take longer sequences as well such as: 1345-6543. This happens to be the formula for the first Hannon exercise (*Hannon* is a famous method-book of piano exercises). The advantage of using the cycle diagram is that it makes it easy to apply the same formula to any scale – not just the major scale.

Pentatonic Scales

Cycle diagrams are ideal for working on pentatonic scales. The left diagram is a C major pentatonic or A minor pentatonic (depending where you start), and the right diagram is the A minor 6th pentatonic or C major #11 (lydian) pentatonic.

You can use these diagrams with any melodic sequence formula such as 1324, which would start with C-E-D-G for the 1st diagram and C-E-D-F# for the 2nd diagram, and then move the structure in either direction to get the full sequence.

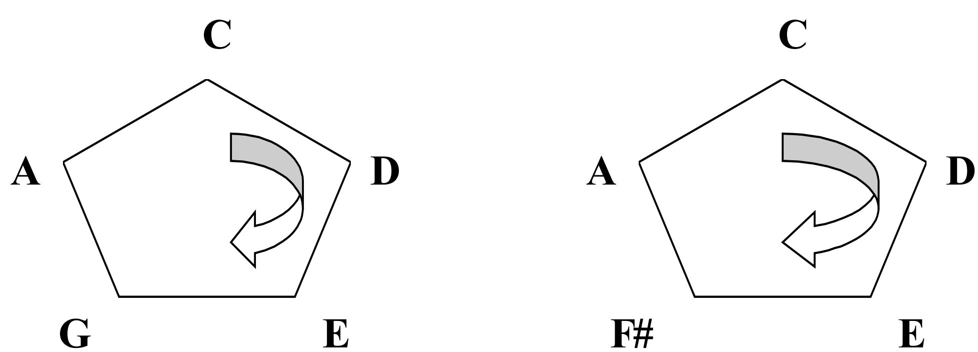

Figure 10: C Major and A Minor-6 Pentatonic Cycle Diagrams

If you start from any note and skip every other note 3 times in a row, you will get all 4-voice pentatonic chords in diatonic 4ths (e.g. E-A-D-G).

Conclusion

There are many mathematical principles embedded in Music. Finding them on your own can be fun and challenging, and can lead you to find simple organizing principles which can replace memorization and increase your understanding of music and give you a great framework for practicing.

Cycles represent fundamental mathematical relationships that exist in music, and mastering them will help you become a better musician.

Using cycle diagrams can help you gain a much deeper underatanding and appreciation of scales than you would by just learning the fingerings in multiple positions.

Acknowledgement

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